The mathematics behind next generation bone prostheses

Dr Vivien Challis

Joint work with Dr Andrew Cramer & A/Prof Tony Roberts
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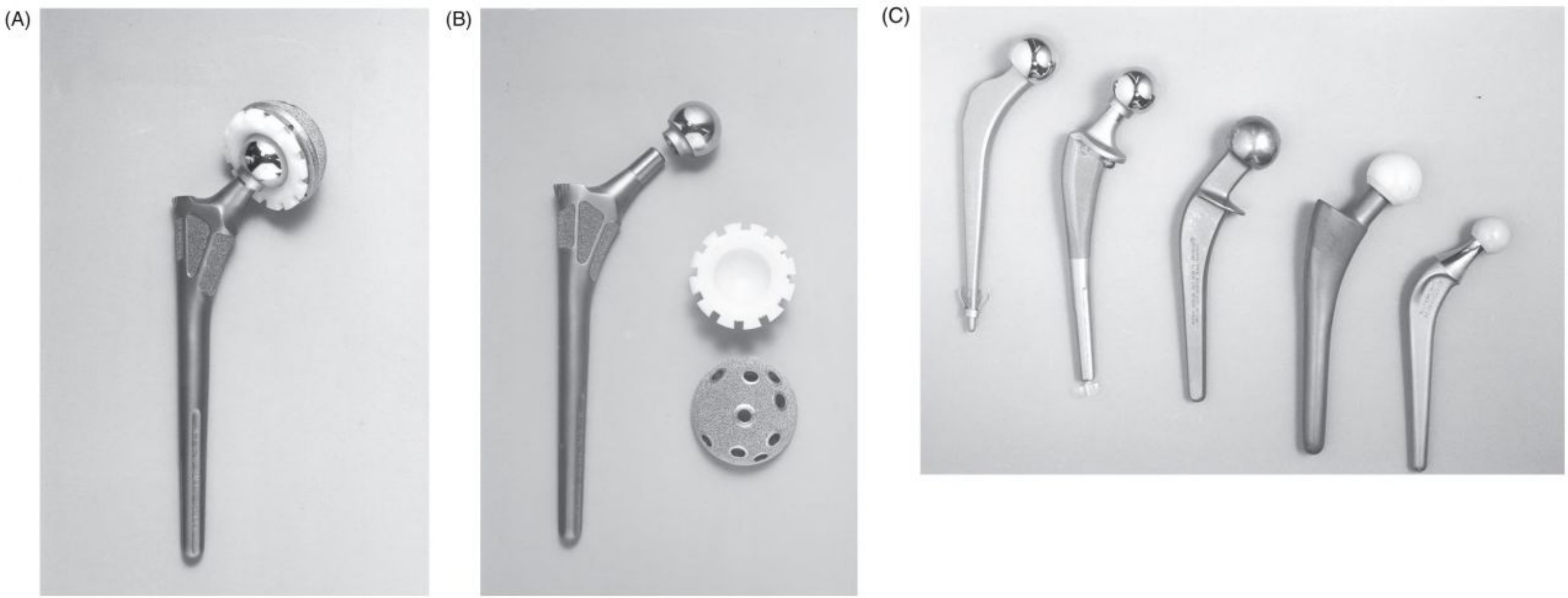
The University of Queensland
Australia

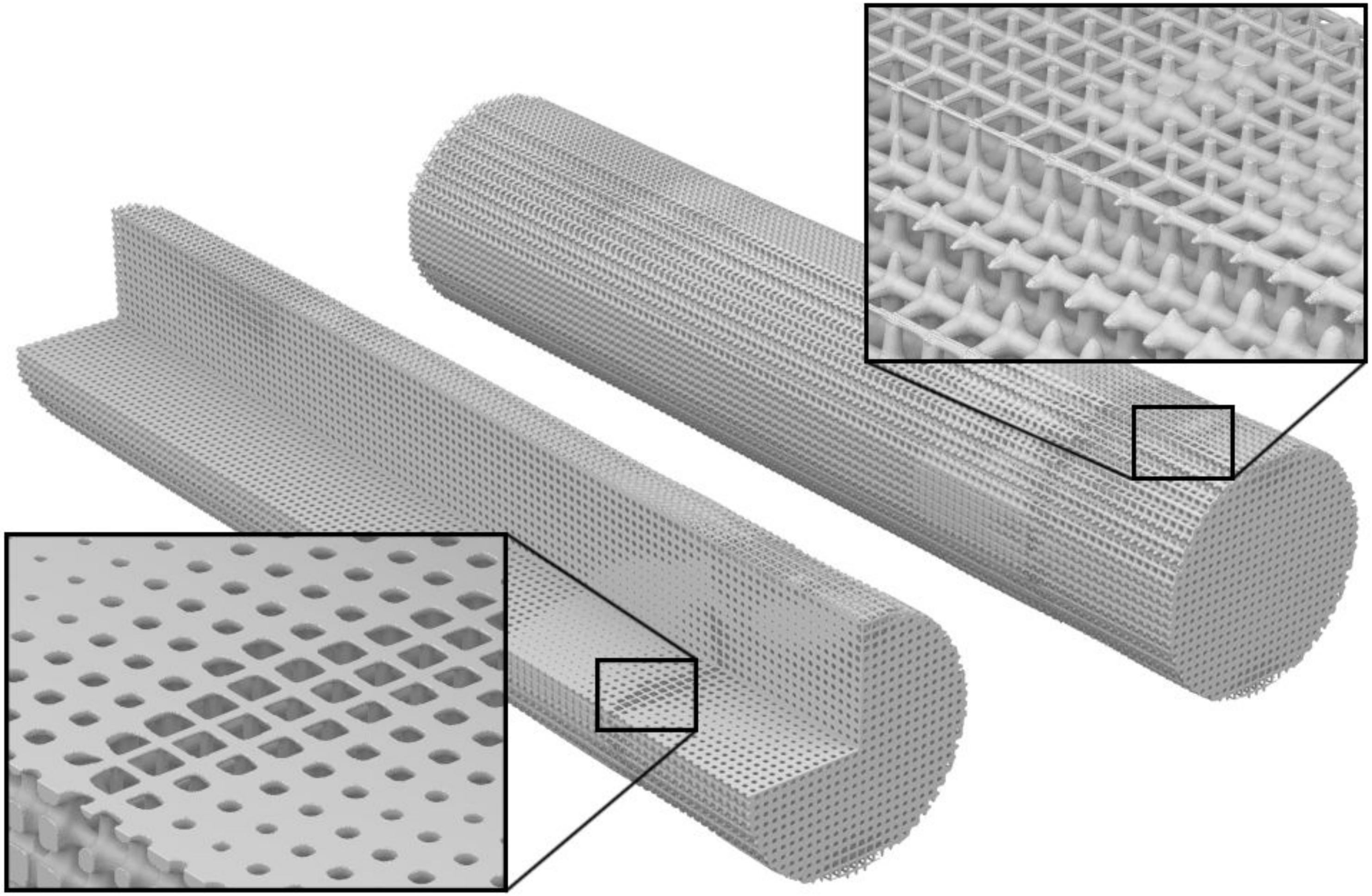
Create change

Femoral bone implants

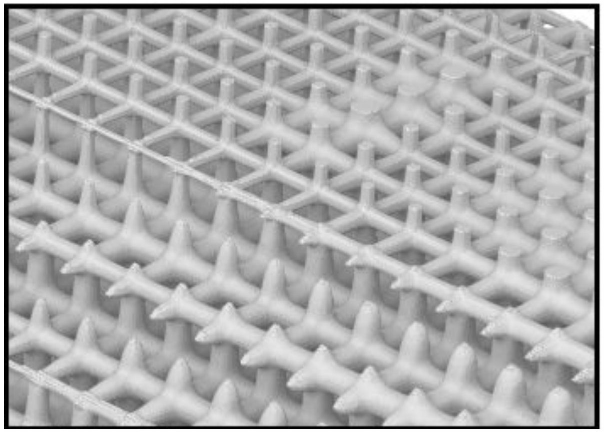


From Benjamin JB, Lund PJ. Orthopedic devices. In: Hunter TB, Bragg DG, eds. Radiologic Guide to Medical Devices and Foreign Bodies, St Louis, MO: Mosby-Year Book, 1994, 348-385.

Looking to the future: Porous implants?



A multiscale design problem



Microstructure requirements

- Porous labyrinths for bone in-growth
- Mechanical stiffness & strength
- Manufacturability

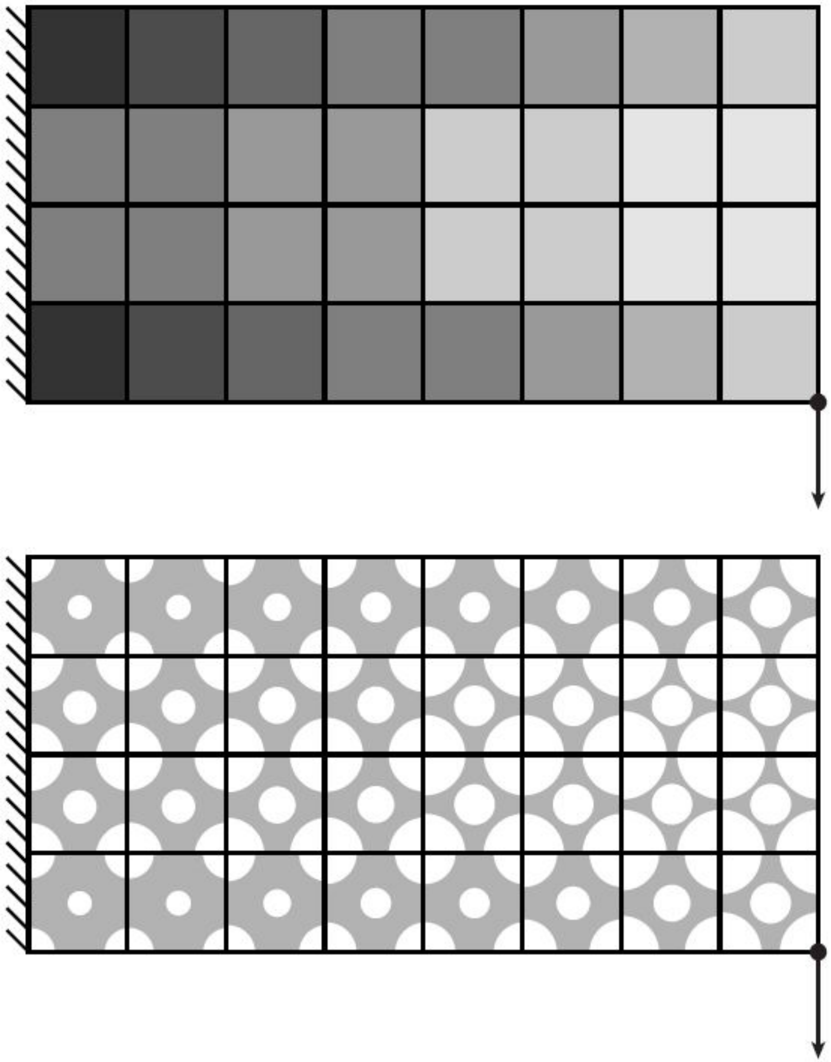
Macroscopic requirements

- Overall shape & size
- Mechanical stiffness and strength, including fatigue
- Minimal bone resorption
- Reduced shear stress along bone/implant interface
- Connectivity of microstructures

Our approach

Design the microstructure family first.

Each element in the macroscopic design uses a microstructure from our set.



Where is the mathematics?

Equations of solid mechanics

- Derivatives
- Integrals
- Simultaneous equations

displacement

$$\epsilon = \frac{\mathrm{d}u}{\mathrm{d}x}$$

strain

Young's modulus

$$\sigma = E\epsilon$$

stress

body forces

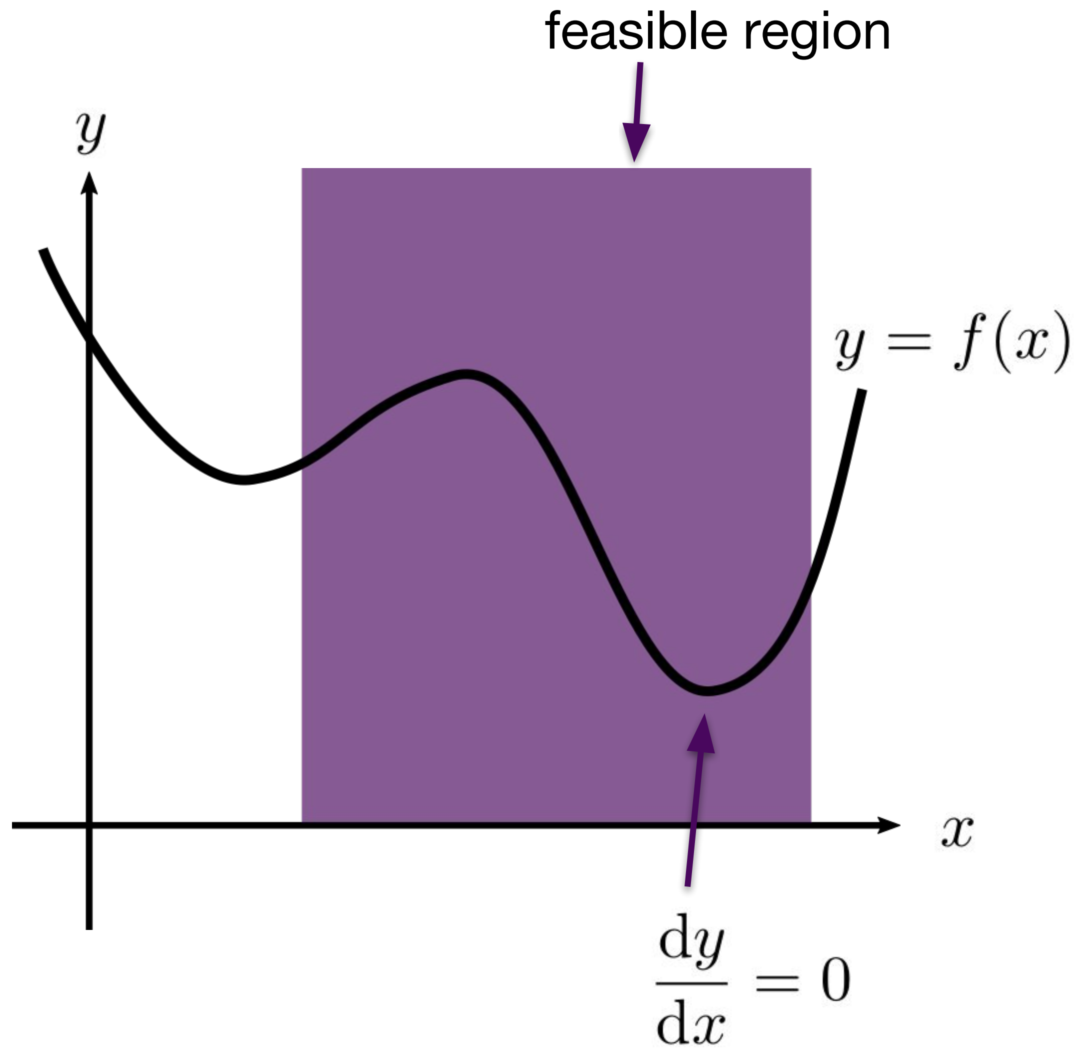
$$\frac{\mathrm{d}\sigma}{\mathrm{d}x} + b = 0$$

spatial coordinate

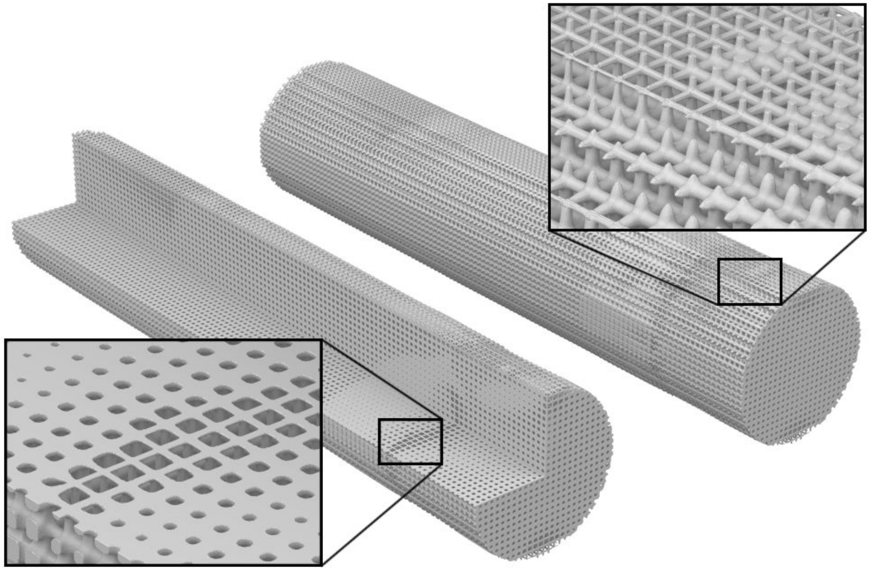
Where is the mathematics?

Optimisation

- Derivatives
- Minimisation
- Feasible region



A sense of scale



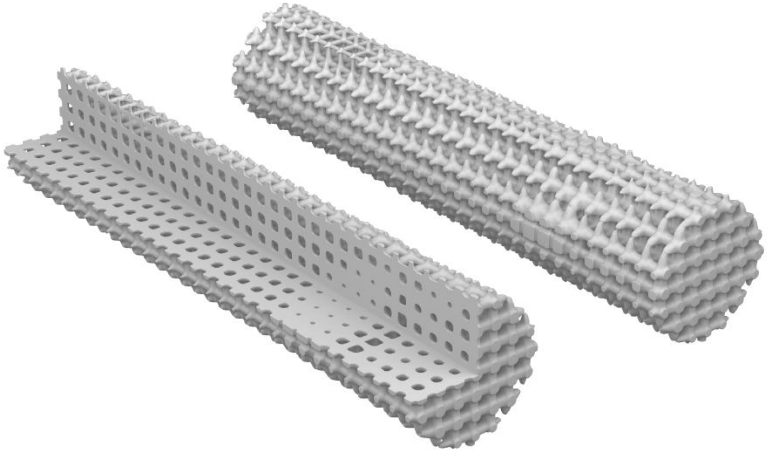
Microstructure design

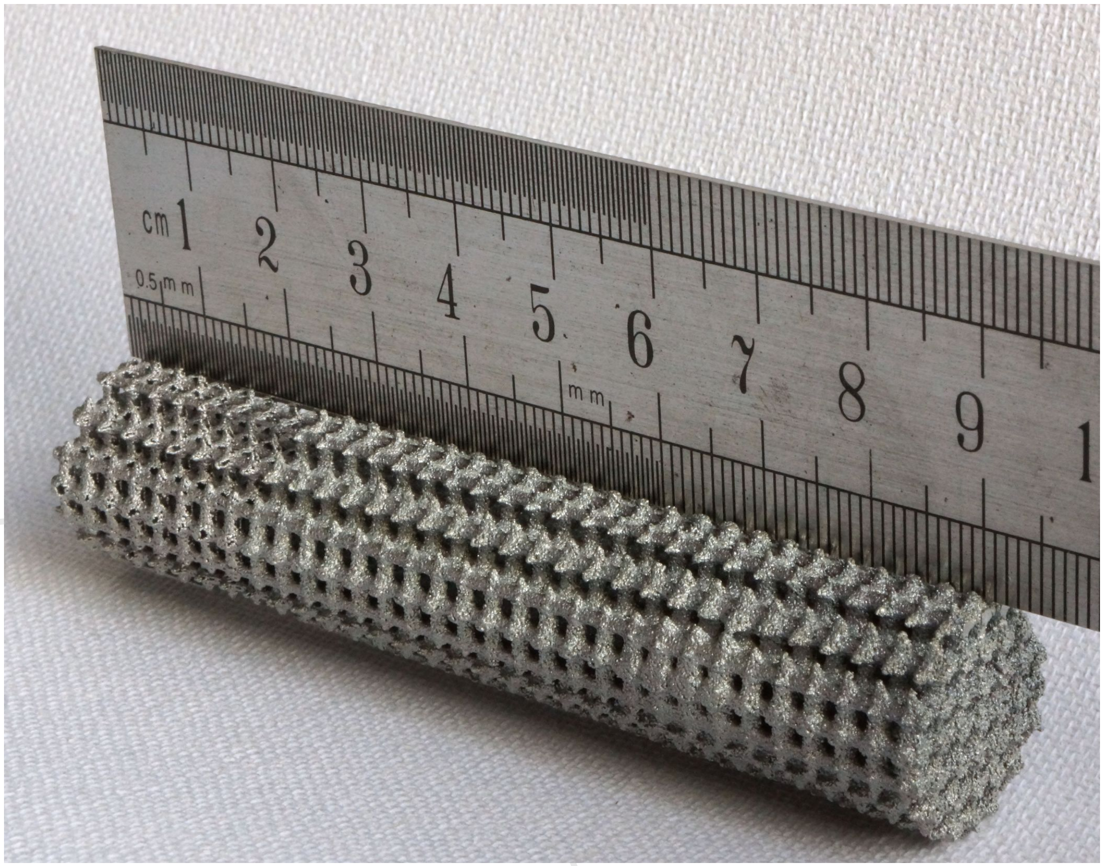
- Matrix equations
  ≈ 780,000 unknowns
- Minimisation
  ≈ 260,000 variables

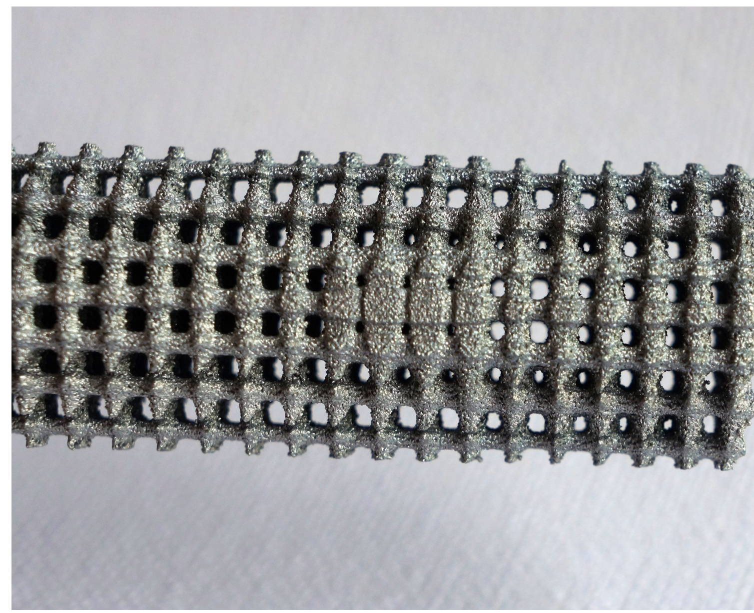
Macroscopic optimisation

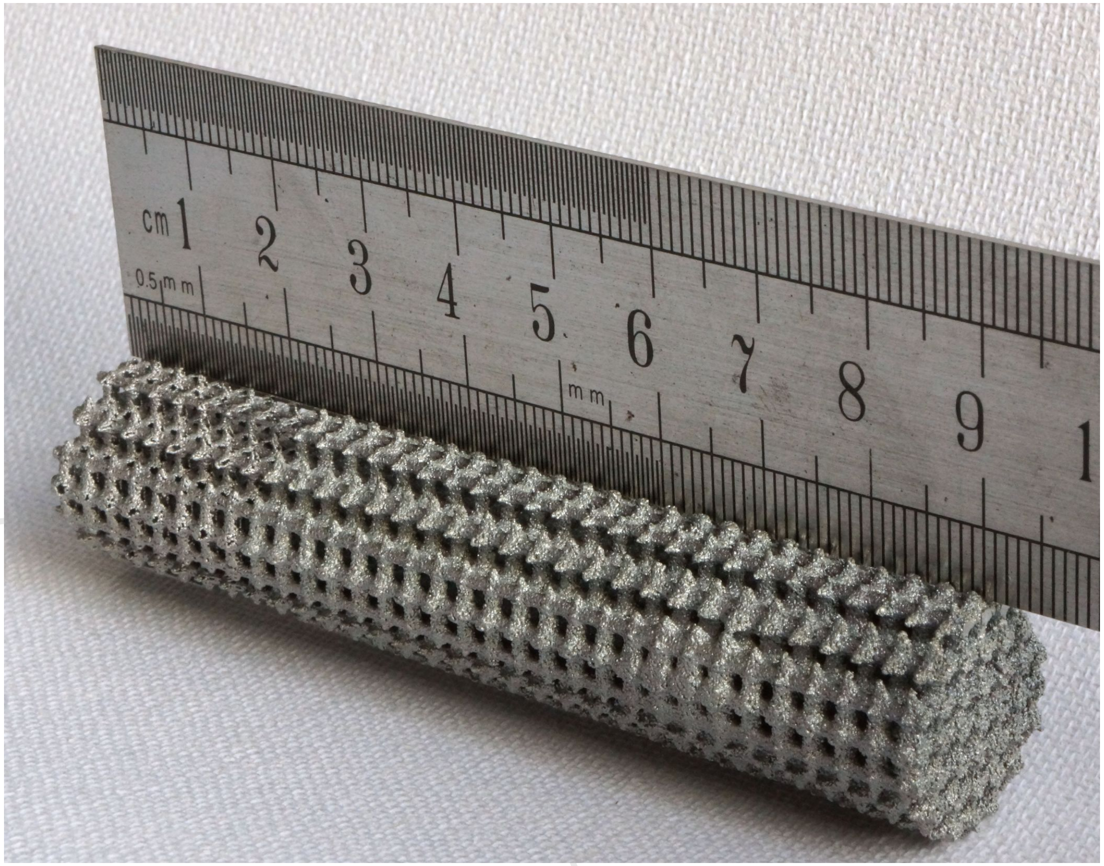
- Matrix equations ≈ 16 million unknowns
- Minimisation ≈ 2 million variables

3D-printed prototype









Dr Vivien Challis
School of Mathematics & Physics
The University of Queensland
vchallis@maths.uq.edu.au
people.smp.uq.edu.au/VivienChallis/

The University of Queensland
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Macroscopic design

- Minimise shear stress on Π
- Subject to constrained bone resorption (5 %)

$$F = \int_{\Pi} (\tau_i \tau_i)^m \, dS = \int_{\Pi} \tau^{2m} \, dS$$

$$m_{\mathrm{r}} = \frac{1}{|\Omega_{\mathrm{B}}|} \int_{\Omega_{\mathrm{B}}} H((1-s)U_{\mathrm{ref}} - U)\rho \, d\boldsymbol{x}$$