Creating impact on the learning of Fractions and Decimals

Anna Bock: anna.bock@amsi.org.au

CHOOSEMATHS

AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

Fractions

CHOOSEMATHS

Aims

- This session will explore research into the teaching and learning of Fractions and Decimals and how we can identify student misconceptions and address them.

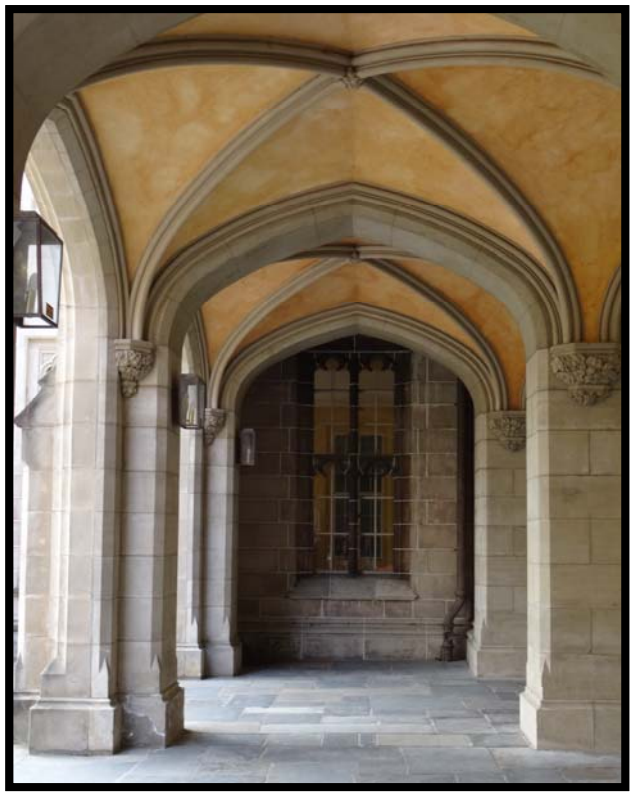
- Participants will also have the opportunity to engage in rich open tasks which provide access to learning of these concepts for all students.

- Fractions and Decimals are often viewed as complex concepts for students to learn and for teachers to teach.
- Using a range of manipulatives, rich open tasks and asking the right questions educators can support students in their struggle to conceptually understand these content areas and impact on their learning.

Who are we?

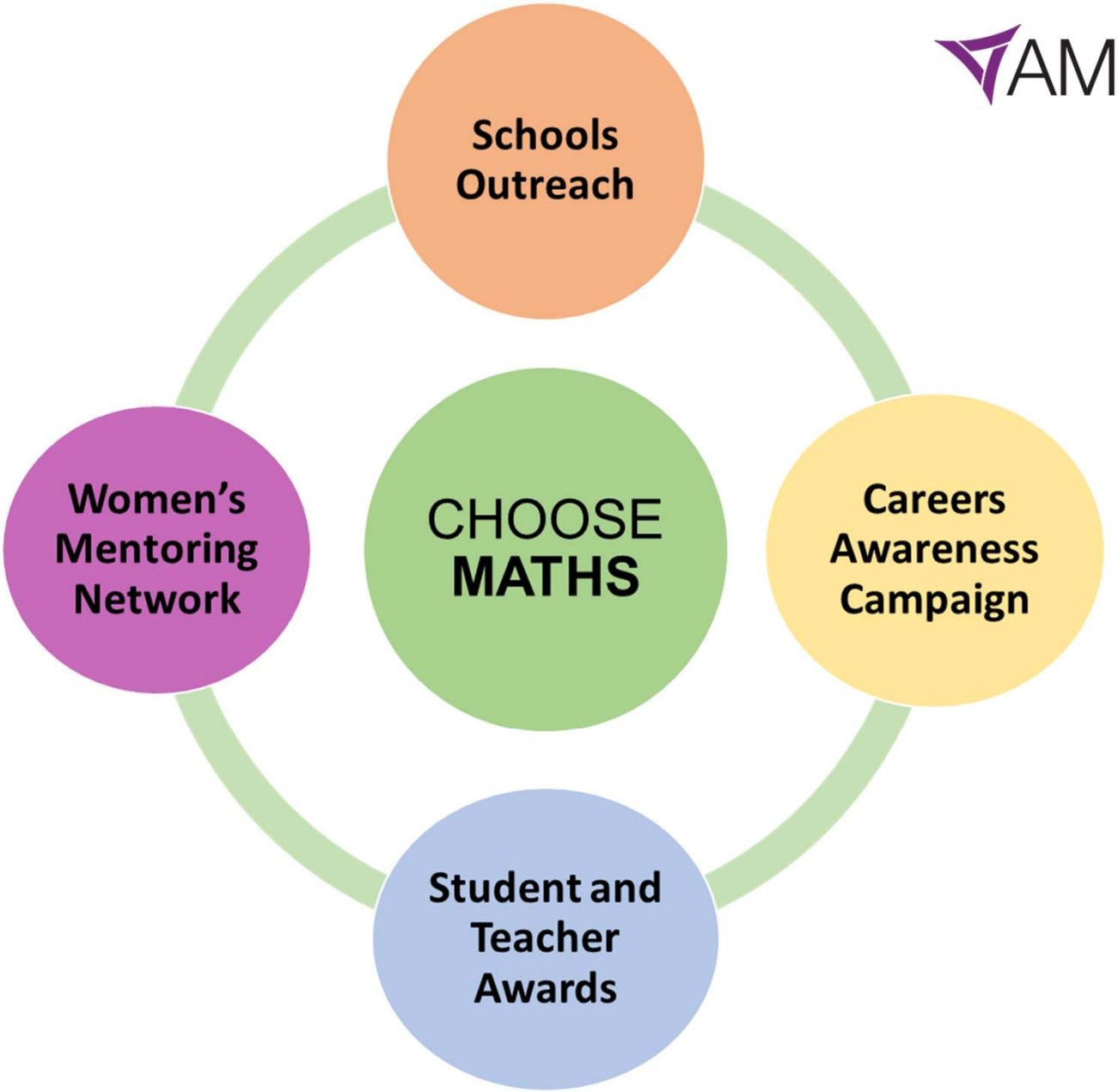
AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

Where are we?



CHOOSEMATHS

AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE



CHOOSEMATHS

https://choosemaths.org.au/

The Australian Curriculum

Mathematics Scope and Sequence: Foundation to Year 6

AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

acara AUSTRALIAN CURRICULUM, ASSESSMENT AND REPORTING AUTHORITY

		Foundation Year	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Number and Algebra	Number and place value	Establish understanding of the language and processes of counting by naming numbers in sequences, initially to and from 20, moving from any starting point Connect number names, numerals and quantities, including zero, initially up to 10 and then beyond Subitise small collections of objects Represent practical situations to model addition and sharing Compare, order and make correspondences between collections, initially to 20, and explain reasoning	Develop confidence with number sequences to and from 100 by ones from any starting point. Skip count by twos, fives and tens starting from zero Recognise, model, read, write and order numbers to at least 100. Locate these numbers on a number line Count collections to 100 by partitioning numbers using place value Represent and solve simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts	Investigate number sequences, initially those increasing and decreasing by twos, threes, fives and ten from any starting point, then moving to other sequences. Recognise, model, represent and order numbers to at least 1000 Group, partition and rearrange collections up to 1000 in hundreds, tens and ones to facilitate more efficient counting Explore the connection between addition and subtraction Solve simple addition and subtraction problems using a range of efficient mental and written strategies Recognise and represent multiplication as repeated addition, groups and arrays Recognise and represent division as grouping into equal sets and solve simple problems using these representations	Investigate the conditions required for a number to be odd or even and identify odd and even numbers Recognise, model, represent and order numbers to at least 10 000 Apply place value to partition, rearrange and regroup numbers to at least 10 000 to assist calculations and solve problems Recognise and explain the connection between addition and subtraction Recall addition facts for single-digit numbers and related subtraction facts to develop increasingly efficient mental strategies for computation Recall multiplication facts of two, three, five and ten and related division facts Represent and solve problems involving multiplication using efficient mental and written strategies and appropriate digital technologies	Recall multiplication facts up to 10 _ 10 and related division facts Investigate and use the properties of odd and even numbers Recognise, represent and order numbers to at least tens of thousands Apply place value to partition, rearrange and regroup numbers to at least tens of thousands to assist calculations and solve problems Investigate number sequences involving multiples of 3, 4, 6, 7, 8, and 9 Develop efficient mental and written strategies and use appropriate digital technologies for multiplication and for division where there is no remainder	Identify and describe factors and multiples of whole numbers and use them to solve problems Use estimation and rounding to check the reasonableness of answers to calculations Solve problems involving multiplication of large numbers by one- or two-digit numbers using efficient mental, written strategies and appropriate digital technologies Solve problems involving division by a one digit number, including those that result in a remainder Use efficient mental and written strategies and apply appropriate digital technologies to solve problems	Identify and describe properties of prime, composite, square and triangular numbers Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers Investigate everyday situations that use integers. Locate and represent these numbers on a number line
	Fractions and decimals		Recognise and describe one-half as one of two equal parts of a whole.	Recognise and interpret common uses of halves, quarters and eighths of shapes and collections	Model and represent unit fractions including 1/2, 1/4, 1/3, 1/5 and their multiples to a complete whole	Investigate equivalent fractions used in contexts Count by quarters halves and thirds, including with mixed numerals. Locate and represent these fractions on a number line Recognise that the place value system can be extended to tenths and hundredths. Make connections between fractions and decimal notation	Compare and order common unit fractions and locate and represent them on a number line Investigate strategies to solve problems involving addition and subtraction of fractions with the same denominator Recognise that the place value system can be extended beyond hundredths Compare, order and represent decimals	Compare fractions with related denominators and locate and represent them on a number line Solve problems involving addition and subtraction of fractions with the same or related denominators Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies Add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers Multiply decimals by whole numbers and perform divisions by non-zero whole numbers where the results are terminating decimals, with and without digital technologies Multiply and divide decimals by powers of 10 Make connections between equivalent fractions, decimals and percentages
	Real numbers	This sequence ends at Year 7						

The Australian Curriculum

Mathematics Scope and Sequence: Year 6 to Year 10

AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

acara AUSTRALIAN CURRICULUM, ASSESSMENT AND REPORTING AUTHORITY

		Year 6	Year 7	Year 8	Year 9	Year 10	Year 10 A
Number and Algebra	Number and place value	Identify and describe properties of prime, composite, square and triangular numbers Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving all four operations with whole numbers Investigate everyday situations that use positive and negative whole numbers and zero. Locate and represent these numbers on a number line	Investigate index notation and represent whole numbers as products of powers of prime numbers Investigate and use square roots of perfect square numbers Apply the associative, commutative and distributive laws to aid mental and written computation Compare, order, add and subtract integers	Use index notation with numbers to establish the index laws with positive integral indices and the zero index Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies	This sequence ends at this year level		
	Fractions and decimals	Compare fractions with related denominators and locate and represent them on a number line Solve problems involving addition and subtraction of fractions with the same or related denominators Find a simple fraction of a quantity where the result is a whole number, with and without digital technologies Add and subtract decimals, with and without digital technologies, and use estimation and rounding to check the reasonableness of answers Multiply decimals by whole numbers and perform divisions that result in terminating decimals, with and without digital technologies Multiply and divide decimals by powers of 10 Make connections between equivalent fractions, decimals and percentages	This sequence ends at Year 6				
	Real numbers	This sequence starts at Year 7	Compare fractions using equivalence. Locate and represent positive and negative fractions and mixed numbers on a number line Solve problems involving addition and subtraction of fractions, including those with unrelated denominators Multiply and divide fractions and decimals using efficient written strategies and digital technologies Express one quantity as a fraction of another, with and without the use of digital technologies Round decimals to a specified number of decimal places Connect fractions, decimals and percentages and carry out simple conversions Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies. Recognise and solve problems involving simple ratios	Investigate terminating and recurring decimals Investigate the concept of irrational numbers, including π Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies Solve a range of problems involving rates and ratios, with and without digital technologies	Solve problems involving direct proportion. Explore the relationship between graphs and equations corresponding to simple rate problems Apply index laws to numerical expressions with integer indices Express numbers in scientific notation		Define rational and irrational numbers and perform operations with surds and fractional indices Use the definition of a logarithm to establish and apply the laws of logarithms

AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

Show me everything you know about $\frac{3}{4}$

CHOOSEMATHS

Questions

Can you show me how you write $\frac{3}{4}$?

Can you show another way?

Can you write it as a number?

Can you write it as a word?

Can you draw it as a quantity?

As an area model?

As a collection ?

Can you show it on a number line?

Meaning of the numbers

- The denominator tells us what is being counted. It tells us **how big the part is. The name and the size of the part.**

- The numerator is the counting number. It tells us **how many parts we have**.

- Think of the fraction $\frac{3}{4}$. The denominator represents how big the part is, i.e. quarters. The numerator is how many of those parts, i.e. 3 (Van de Walle et al. 2010).

Improper fractions

- A fraction whose numerator is greater than its denominator.
- The introduction of improper fractions should occur as early as possible.

- What would 5 quarters look like?

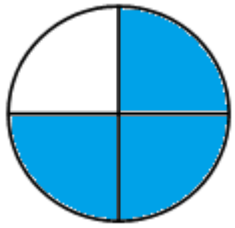
Fractional Language

- Use of Fractional language will support students with their understandings of both Fractions and Decimals.
- With the Fraction $\frac{3}{4}$ describing it as 3 out of 4 or 3 over 4 causes confusion for students in making sense of Fractions as a number.
- Many students find it difficult to understand improper Fractions as these descriptions make it difficult to go beyond a whole.

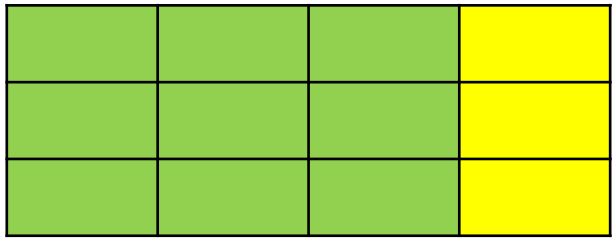
" Be sure to read decimals aloud in a way that emphasises the tie between decimals and fractions. For example, read 24.09 as 'twenty-four and nine hundredths', not as ... 'twenty-four point oh nine'"

Reys, Lindquist, Lambdin & Smith, 2009

Fraction Models



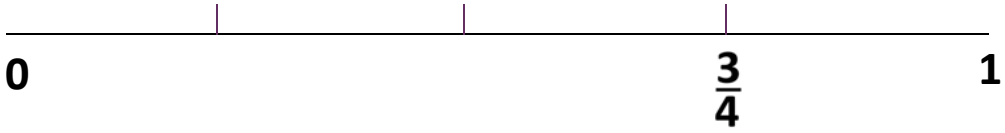
Area Model



Volume Model

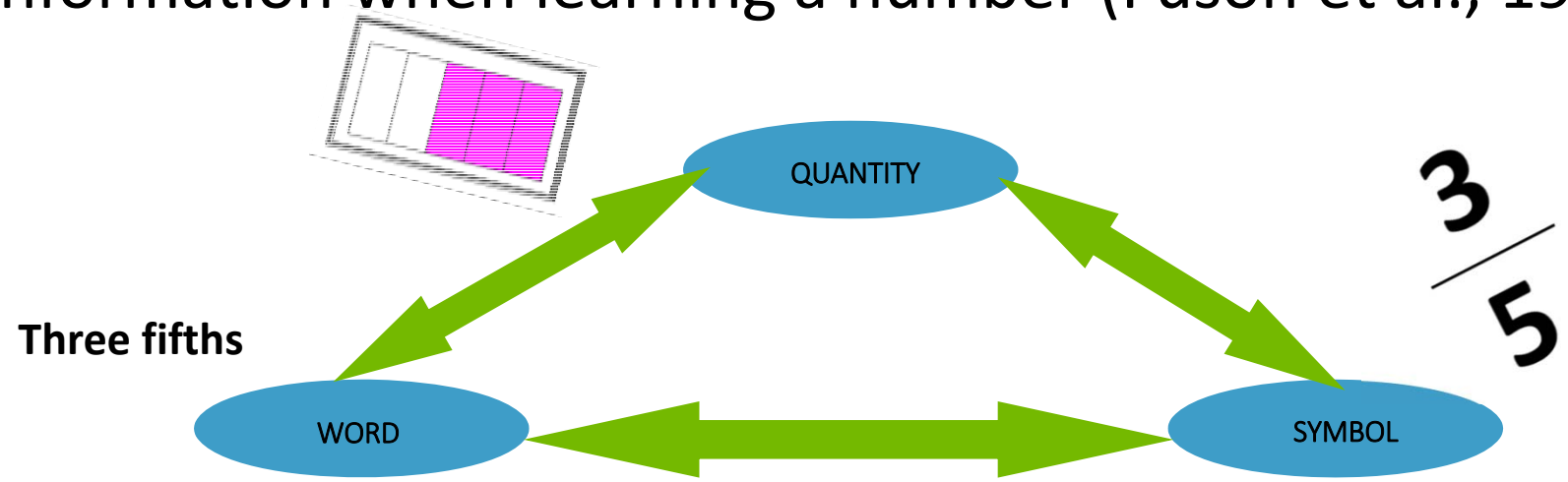
Discrete Model

Number Line



Number Triad Relationship

Relationships between the three important pieces of information when learning a number (Fuson et al., 1997).



Students need to be able to work with each relationship, starting at each and any part of the triad. This triad can be used with fractions, decimals, multi-digit numbers.

Equivalence

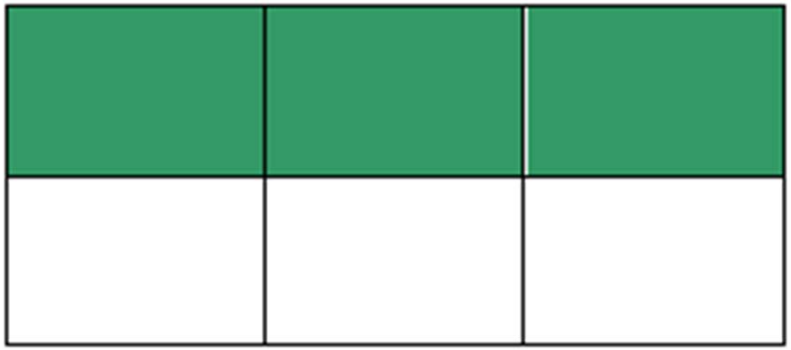
Recognising that fractions can be represented as:

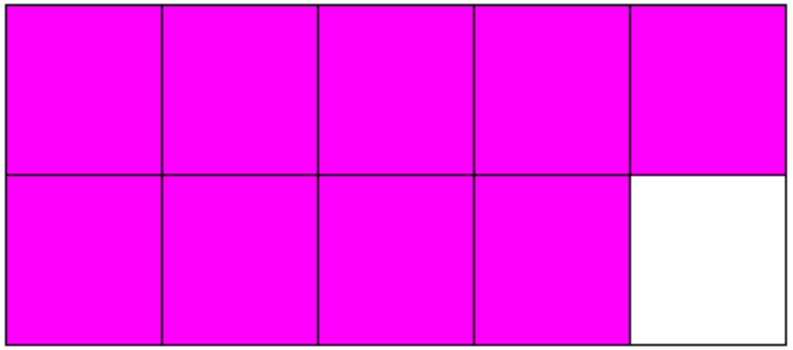
- **Decimals:** $\frac{1}{2}$ is the same as 0.5

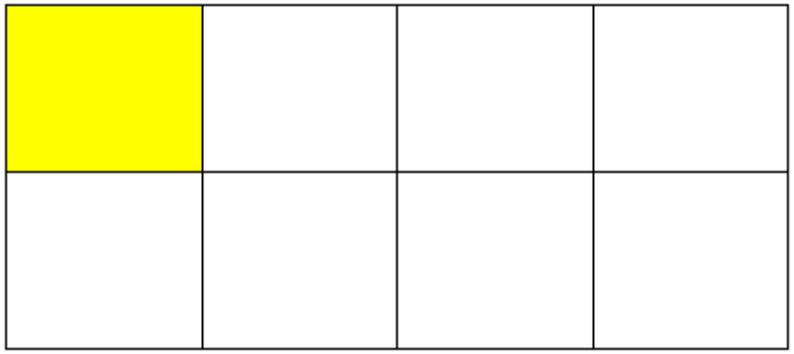
- **Percentages:** $\frac{1}{2}$ is the same as 50%

- **Equivalent fractions:** $\frac{1}{2}$ is the same as $\frac{2}{4}$, $\frac{4}{8}$ etc.

- **Ratios** $\frac{1}{2}$ is the same as 1:2



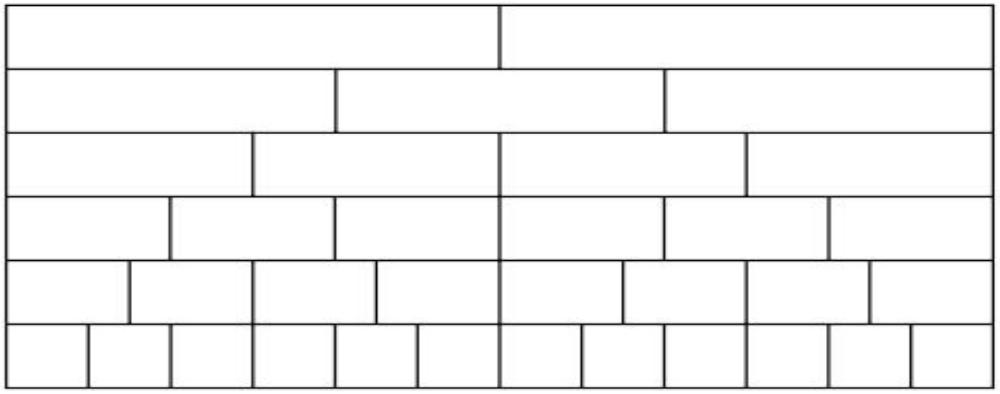




Clarke & Roche

Activity Sheet: Colour in Fractions

AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

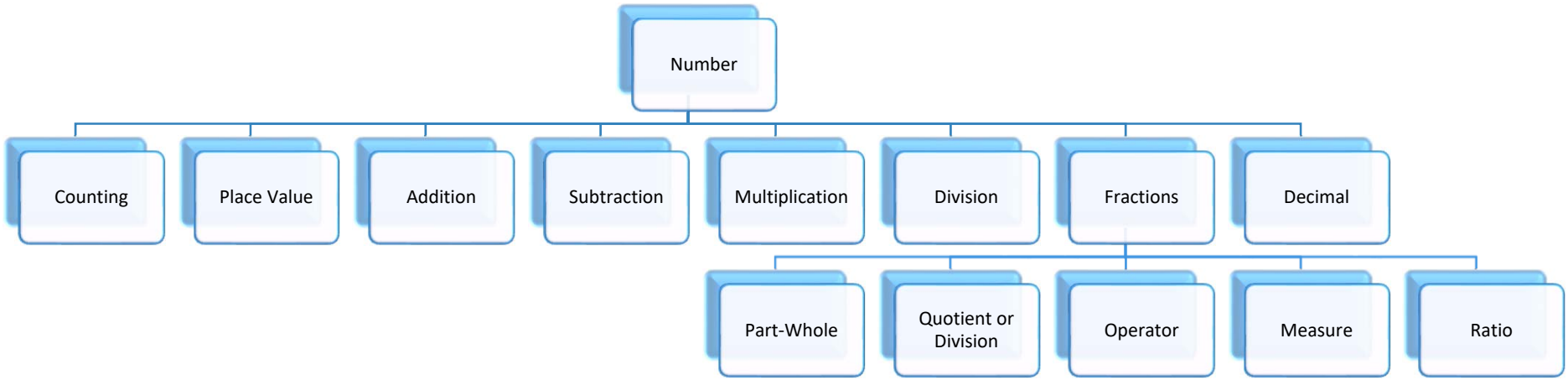


What I rolled	What I shaded

What I rolled	What I shaded

CHOOSE**MATHS**

The constructs that fractions represent



Fraction as Part-Whole

- Partitioning either a continuous quantity (including area, length and volume models) or a set of discrete objects into equal sized subparts or sets.

- For example, $\frac{3}{4}$ is interpreted as 3 parts out of 4 equal parts.

Fraction as Division

- A fraction $\frac{a}{b}$ may also represent the operation of division or the result of a division such that $3 \div 4 = \frac{3}{4}$.
- The division interpretation may be understood through partitioning and equal sharing.

For example, 3 chocolate bars shared between 4 people

Clarke, D., 2006. Fractions as Division: The forgotten notion?. *Australian Primary Mathematics Classroom,* 11(3), pp. 4-10

Fraction as operator

- A fraction can be used as an operator to find an amount of a collection or a quantity.

- For example, $\frac{3}{4}$ of 12 chocolates = 9 chocolates

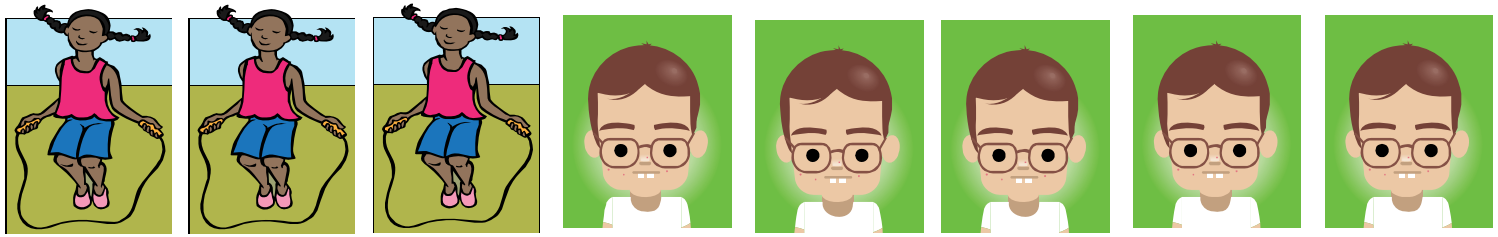
  or $\frac{5}{3}$ x 1.5 metres = 2.5 metres.

Fraction as ratio

- Fractions can be used as a method of comparing the sizes of two sets or two measurements to either the **whole** or to **another set**, e.g. in a class of 3 girls and 5 boys:

  - the ratio of **girls to the class is 3:8 (part-whole), which is equivalent to** $\frac{3}{8}$ **of the class being girls**

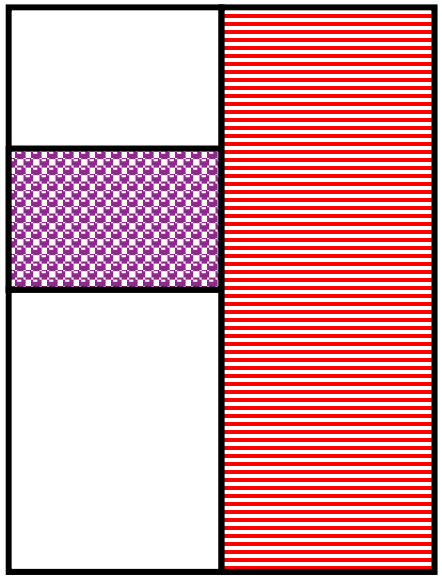
  - the ratio of **girls to boys is 3:5 (part-part), which is equivalent to the girls being** $\frac{3}{5}$ **of the boys.**



If the dotted (purple) rectangle represents $\frac{2}{3}$

what fraction is represented by the striped (red) rectangle?

Work out the answer in two different ways.



AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

Decimals

CHOOSEMATHS

Decimal Comparison Test

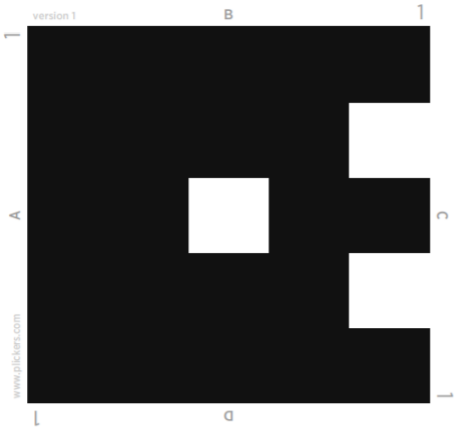
NAME:_____

For each pair of decimal numbers, circle the one which is LARGER.

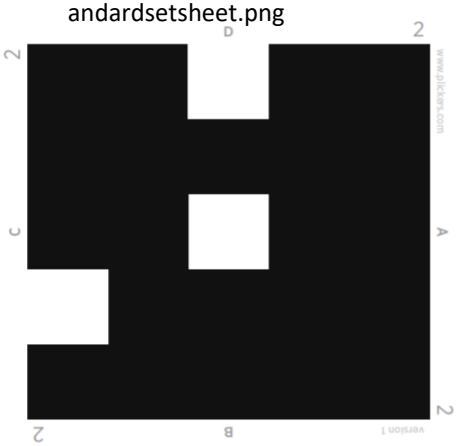
4.8	4.63
0.5	0.36
0.75	0.8
0.37	0.216
3.92	3.4813
5.62	5.736
0.6	0.85
0.426	0.3
2.516	2.8325
7.942	7.63
4.08	4.7
1.85	1.84
17.353	17.35

Plickers

- Plickers are an easy to use tool to collect information
- Look at the Plicker card and see the number and the different letters
- Turn your card so your response is at the top
- Try to hold the card in the bottom corner so the image is not covered
- Be honest – your response is anonymous



https://www.plickers.com/assets/images/847856ab.standardsetsheet.png



CHOOSEMATHS

More examples		Pattern 1	Pattern 2	Pattern 3
		Apparent Expert (A)	Longer-is-Larger (L)	Shorter-is-Larger (S)
4.8	4.63	✓	X	✓
0.5	0.36	✓	X	✓
0.8	0.75	✓	X	✓
0.37	0.216	✓	X	✓
3.92	3.4813	✓	X	✓
5.736	5.62	✓	✓	X
0.75	0.5	✓	✓	X
0.426	0.3	✓	✓	X
2.8325	2.516	✓	✓	X
7.942	7.63	✓	✓	X

MAV Conference 2004

Decimal Misconceptions

AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

Longer is larger…

- **Whole number thinking –**

  4.63 is larger than 4.8 as 63 is greater than8.

- **Right hand overflow thinking –**

  4.63 is greater than 4.8 as 63 tenths is greater than 8 tenths.

Money Thinkers…

- Have a good understanding of the first two decimal places and may view decimals as two numbers separated by a dot, the first possibly representing dollars and the second cents.

- Avoid teaching decimals through money.

Decimal Misconceptions

Shorter is larger…

- **Denominator focused thinking :**

  incorrectly generalising the fact that 1 tenth is bigger than 1 hundredth to ‘any number of tenths is bigger than any number of hundredths, e.g. 0.4 is greater than 0.83.

- **Reciprocal thinking :**

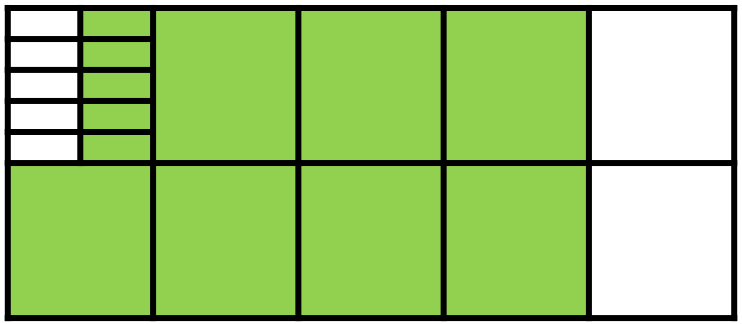
  incorrectly connects decimals with fractions, e.g. perceives 0.3 as 3 parts rather than 10 equal parts and as such 0.3 is larger than 0.4 because $\frac{1}{3}$ is greater than $\frac{1}{4}$.

- **Negative thinking :**

  0.3 is larger than 0.4 as -3 is larger than -4.

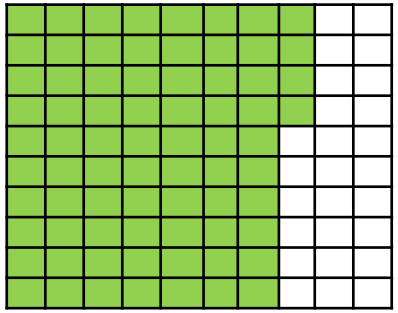
Decimal Models

Decimat

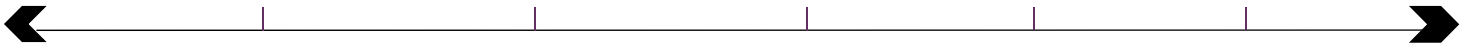


Unifix

Hundred Squares



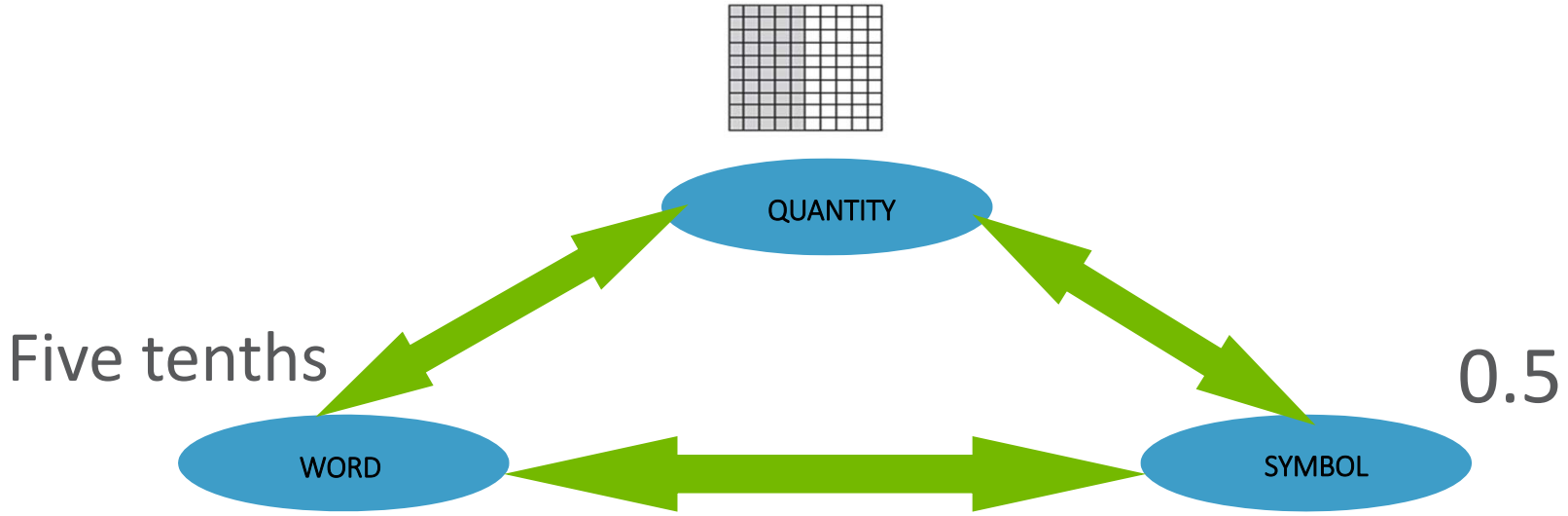
LAB



0 0.75 1 Number Line

Number Triad Relationship

Relationships between the three important pieces of information when learning a number (Fuson et al., 1997).

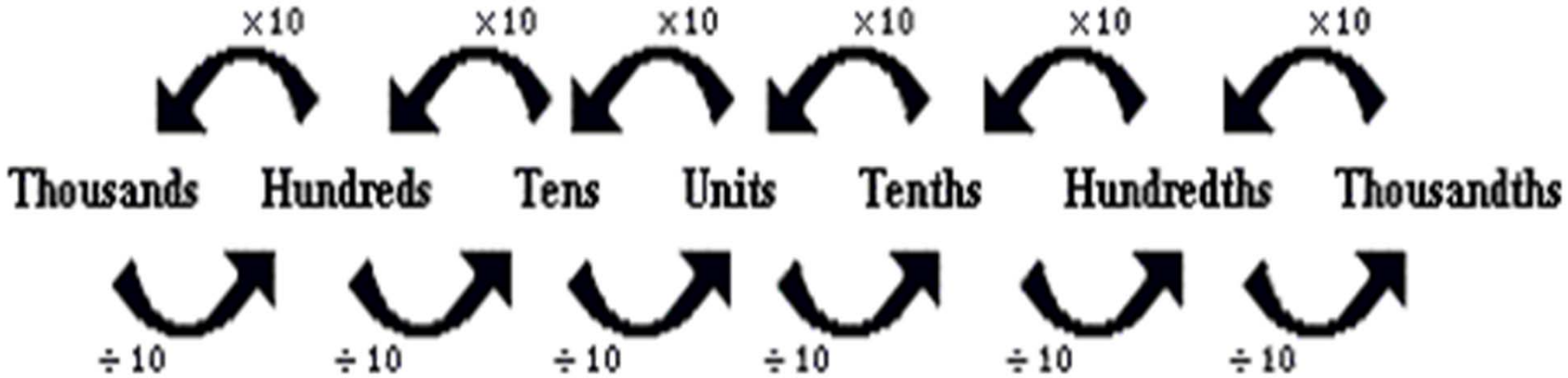


Students need to be able to work with each relationship, starting at each and any part of the triad. This triad can be used with fractions, decimals, multi-digit numbers.

Base 10 system

Recognising that our number system is based on grouping quantities in tens.

Each place has a value that is 10 times greater than the place to is right and one tenth of the value to its left.



Linear Arithmetic Blocks

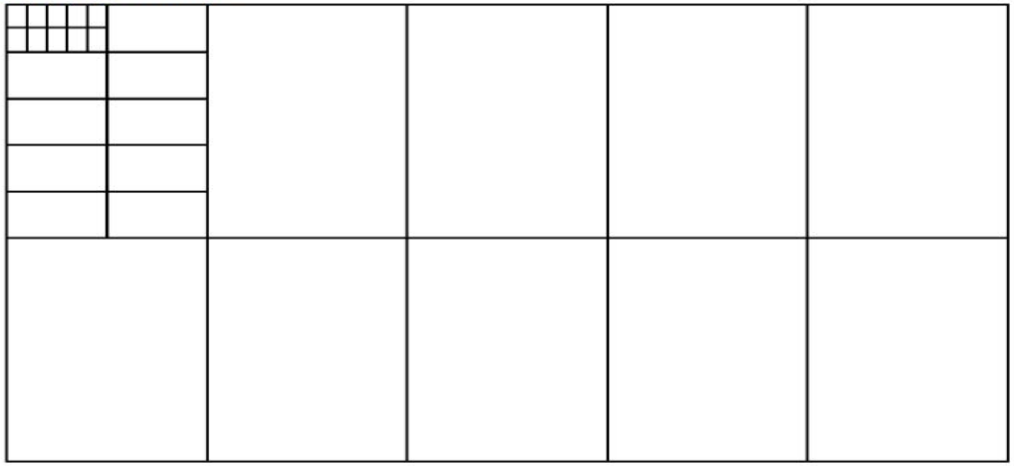
- LAB clearly shows that 0.2 is larger than 0.13 and not the other way around! Many longer-is-larger children think that 0.13 is larger than 0.2 (as 13 is larger than 2).
- It also shows other properties clearly including:
- equivalence of 0.2 and 0.20 (2 tenths and 20 hundredths)
- equivalence of 0.13 (13 hundredths) with 0.1 + 0.03 (one tenth and 3 hundredths)
- density of decimal numbers (that there are other decimals between 0.24 and 0.25 or between 0.247 and 0.248 etc)

(Steinle 2012)

Deciwire

CHOOSE**MATHS**

Introducing Decimats



What I rolled		How much is shaded altogether?	
Fraction	Decimal	Fraction	Decimal

Adapted from the article 'Decimats: Helping Students to Make Sense of Decimal Place Value', by Anne Roche.

AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

Implications for Teaching

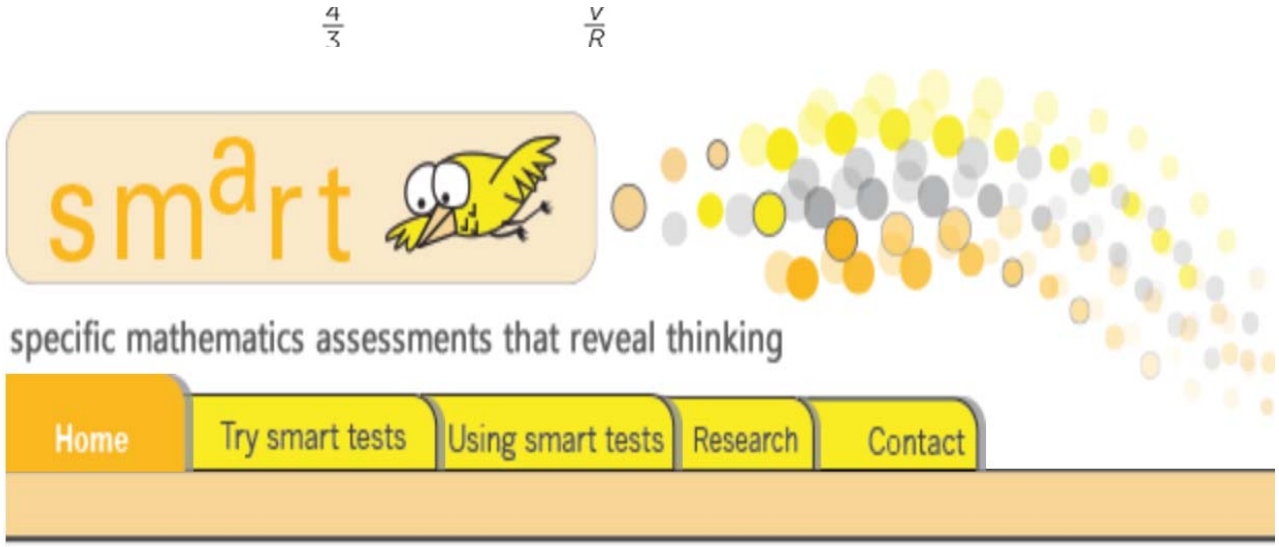
1. Use a decimal comparison test to determine which students hold specific misconceptions.
2. To discourage whole number thinking avoid using equal length decimals in comparisons tasks and classroom activities.
3. Avoid using the rule of “adding zeros” to compare decimals as this promotes the misconception that decimals can be compared in the same way as whole numbers.
4. Use fractional language to describe decimals (e.g., 2.56 as 2 and 56 hundredths), as this helps to promote a better understanding of decimal value.
5. As with fractions, use proportional models and representations to help build an understanding of decimal size.

Reference: Anne Roche for CTLM Program Australian Catholic University (ACU).

AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

www.calculate.org.au

CHOOSEMATHS



Welcome to smart tests

This is the home page of the "SMART::tests" research and development project. The project is being conducted at the University of Melbourne by Professor Kaye Stacey, Dr Vicki Steinle, Ms Beth Price and Dr Eugene Gvozdenko.
A 'smart test' is a **s**pecific **m**athematics **a**ssessment that **r**eveals **t**hinking. These innovative tests, which are accessed through an intelligent on-line environment, provide teachers with an informative diagnosis of their students' conceptual understanding of many of the topics in upper primary and junior secondary school mathematics. Within a few seconds of a student completing a test, the research-backed diagnosis is ready for the teacher. We intend that this information will be concise enough to be readily useable by teachers, deep enough to make a real difference to lesson content, and linked to appropriate teaching resources. Tests can be used by students anywhere there is a computer with an internet connection.

CHOOSE**MATHS**

References

- Catholic Education Office Melbourne, 2013. Key Ideas for Concept Development in Mathematics.
- Clarke, D., 2006. Fractions as Division: The forgotten notion?. *Australian Primary Mathematics Classroom,* 11(3), pp. 4-10.
- Clarke, D. & Roche, A., 2010. The Power of a Single Game to Address a Range of Important Ideas in Fraction Learning. *Australian Primary Mathematics Classroom,* 15(3), p. 18–24.
- Clarke D., Roche, A. & Mitchell, A. (2008) 10 Practical Tips for Making Fractions Come Alive and Make Sense *Teaching Mathematics in Middle School* 13(7) pp. 372 – 380
- DEST, 2004, *Understanding place value: A case study of the base ten game,* Canberra, DEST.
- Reys, R, Lindquist, M, Lambdin, D & Smith, N, 2009, *Helping children learn mathematics*, Wiley, USA,189-190.
- Roche, A. 2005, Longer is Larger, or is it? APMC 10 (3)
- Steinle, V., 2004.Detection and Remediation of Decimal Misconceptions, MAV
- Young-Loveridge, J.; Mills, J.,2011. Deciwire: An Inexpensive Alternative for Constructing Linear Representations of Decimals. *Australian Primary Mathematics Classroom,* 16(3)